Statistics

Lecture 9

Feb 19-8:47 AM

Counting:

There are 5 people

Adam, Bill, Carol, David, Eddie

I need to Select 2 people

~~AB~~	~~AC~~	~~AD~~	~~AE~~
BA	~~BC~~	~~BD~~	~~BE~~
CA	CB	~~CD~~	~~CE~~
DA	DB	DC	~~DE~~
EA	EB	EC	ED

If order does not matter,

10 ways

Use Combination formula

$${}_nC_x = \frac{n!}{x!\,(n-x)!}$$

$${}_5C_2 = \frac{5!}{2!\cdot(5-2)!} = \frac{5\cdot\overset{2}{\cancel{4}}\cdot\cancel{3\cdot2\cdot1}}{\cancel{2}\cdot1\cdot\cancel{3\cdot2\cdot1}} = \frac{5\cdot2}{1} = \boxed{10}$$

How to use TI:

5 $\boxed{\text{Math}}$ $\boxed{\rightarrow}$ PRB ↓ $\boxed{{}_nC_r}$ 2 $\boxed{\text{Enter}}$

Jan 22-4:34 PM

How many ways can we select 3 items from 8 different items when order does not matter and no replacement?

$${}_8C_3 = \frac{8!}{3!\cdot(8-3)!}$$

$$= \frac{8!}{3!\cdot 5!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{3\cdot 2}\cdot 1\cdot \cancel{5!}}$$

$$= \frac{56}{\cancel{1}} = \boxed{56}$$

$8\ \boxed{\text{Math}} \rightarrow \text{PRB} \downarrow$

$\boxed{{}_nC_r}\ 3\ \boxed{\text{Enter}}$

Jan 22-4:42 PM

CA Lotto

Choose 5 numbers in any order from 1 to 50.

How many combinations?

$${}_{50}C_5 = \boxed{2,118,760}$$

Jan 22-4:46 PM

You are taking a quiz. There are 10 True/False questions. You are making random guesses.

How many ways can you have 7 correct guesses?

$${}_{10}C_7 = \boxed{120}$$

$CCCCCCC\bar{C}\bar{C}\bar{C}$

⋮

How about 5 correct answers?

$${}_{10}C_5 = \boxed{252}$$

Jan 22-4:48 PM

SG16

Binomial Prob. Dist.

1) n independent events.

2) Each event has only two outcomes.

$P(\text{Success}) = p$ $P(\text{Failure}) = q$

$$p + q = 1$$
$$q = 1 - p$$

p & q do not change for all events.

3) x is # of successes,
$n-x$ is # of failures.

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

# of ways to have x successes.

Jan 22-4:52 PM

You flip a fair coin 10 times. $n=10$

Success is to land tails. $P=.5$

$q=.5$

P(lands exactly 7 tails)

$P(x=7) = {}_{10}C_7 \cdot (.5)^7 \cdot (.5)^3 = \boxed{.117}$

Jan 22-4:57 PM

You are taking a multiple-choice quiz.

You are making random guesses. $q=\frac{3}{4}=.75$

There are $\boxed{\text{12 questions}}$. $n=12$ $P=\frac{1}{4}=.25$

Each question has 4 choices but only one correct choice.

P(guess exactly 5 Correct Answers)

$P(x=5) = {}_{12}C_5 \cdot (.25)^5 \cdot (.75)^7 = \boxed{.103}$

Using TI Command

[2nd] [VARS] ↓ ↓ ··· [binompdf(]

Trials : 12 12,.25,5

P : .25

x value : 5

[Paste] [Enter]

Your Work

$\boxed{P(x=5) = \text{binompdf}(12, .25, 5) = .103}$

Jan 22-5:02 PM

A loaded coin is tossed 20 times.

Success is to land tails.

$n=20$

$P=.6$

Prob. of landing tail on any toss is .6.

P(lands tails exactly 15 times).

$P(x=15) = \text{binompdf}(20, .6, 15) = \boxed{.075}$

P(lands tails at most 15 times)

$P(x \le 15) = \text{binomcdf}(20, .6, 15) = \boxed{.949}$

Jan 22-5:10 PM

I randomly selected 25 students.

Prob. that each student pass the math class is .8.

$n=25$

$P=.8$

1) P(exactly 10 pass the class)

$P(x=10) = \text{binompdf}(25, .8, 10) = \boxed{1.15 \times 10^{-5}}$

2) P(at most 20 pass the class)

$P(x \le 20) = \text{binomcdf}(25, .8, 20)$

$= \boxed{.579}$

Jan 22-5:17 PM

Consider a True/False exam with 100 questions. You are making random guesses.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $np = 100(.5) = \boxed{50}$

5) $npq = 100(.5)(.5) = \boxed{25}$

6) $\sqrt{npq} = \sqrt{25} = \boxed{5}$

7) P(guess correctly exactly 60 times)

$P(x=60) = \text{binompdf}(100, .5, 60) = \boxed{.011}$

8) P(guess correctly fewer than 60 times)

$P(x < 60) = P(x \leq 59)$

$= \text{binomcdf}(100, .5, 59) = \boxed{.972}$

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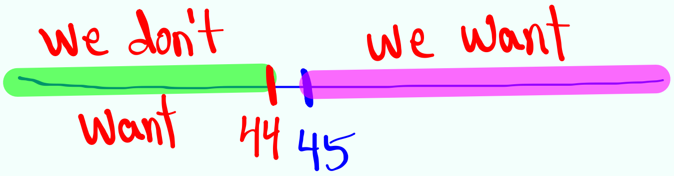
9) P(guess correctly at least 45 times)

$P(x \geq 45) = 1 - P(x \leq 44)$

Total Prob.



$= 1 - \text{binomcdf}(100, .5, 44)$

$= \boxed{.864}$

Jan 22-5:31 PM

400 Voters were randomly Selected.

Prob. that any voter voted Yes on tougher gun control is .8.

1) $n=400$ 2) $P=.8$ 3) $q=.2$

4) $np=400(.8)$ $=\boxed{320}$

5) $npq=400(.8)(.2)$ $=\boxed{64}$

6) $\sqrt{npq}$ $=\sqrt{64}=\boxed{8}$

7) P(exactly 325 voted Yes)

$P(X=325)=\text{binompdf}(400, .8, 325)=\boxed{.042}$

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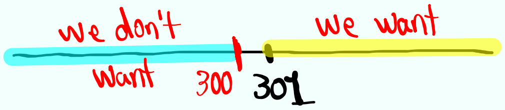
8) P(at most 330 voted Yes)

$P(X\le 330)=\text{binomcdf}(400, .8, 330)$

$=\boxed{.907}$

9) P(more than 300 Voted Yes)

$P(x>300)=P(x\ge 301)$



$=1-P(x\le 300)$

Total Prob.

$=1-\text{binomcdf}(400, .8, 300)$

$=\boxed{.991}$

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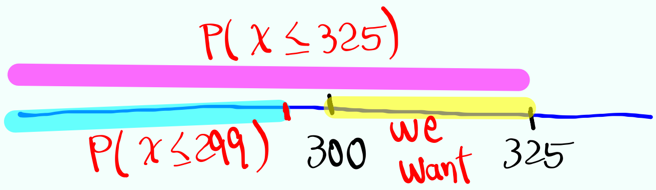
10) P(between 300 and 325 , inclusive voted Yes)



$$P(300 \le x \le 325) = P(x \le 325) - P(x \le 299)$$

$$= \text{binomcdf}(400, .8, 325) -$$

$$\text{binomcdf}(400, .8, 299)$$

$$= \boxed{.746}$$

Jan 22-5:48 PM

Binomal Prob. Dist.

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard deviation $\sigma = \sqrt{\sigma^2}$

Jan 22-6:05 PM

Consider a binomial Prob. dist with
$n=100$ & $p=.5$

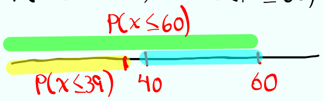
1) $q=1-p=\boxed{.5}$ 2) $\mu=np=100(.5)=\boxed{50}$

3) $\sigma^2=npq=100(.5)(.5)=\boxed{25}$ 4) $\sigma=\sqrt{\sigma^2}=\sqrt{25}=\boxed{5}$

68% Range $\mu\pm\sigma=50\pm5\rightarrow\boxed{45 \text{ to } 55}$

95% Range
Usual Range $\mu\pm2\sigma=50\pm2(5)$
$=50\pm10\rightarrow\boxed{40 \text{ to } 60}$

$P(40\le x\le 60)=P(x\le 60)-P(x\le 39)$



= binomcdf(100, .5, 60) − binomcdf(100, .5, 39)

$=\boxed{.965}$

Jan 22-6:07 PM

Suppose prob. that a newborn baby is a boy is .6.

If we randomly select 250 newborn babies.

1) $n=250$ 2) $p=.6$ 3) $q=.4$

4) $\mu=np=250(.6)=\boxed{150}$ 5) $\sigma^2=npq=250(.6)(.4)=\boxed{60}$

6) $\sigma=\sqrt{\sigma^2}=\sqrt{60}\approx\boxed{8}$

7) Usual Range $\mu\pm2\sigma$
"95% Range" $=150\pm2(8)$
$=150\pm16\rightarrow\boxed{134 \text{ to } 166}$

8) P(we have between 134 and 166 boys, inclusive)

$P(134\le x\le 166)=$ Reduce

binomcdf(250, .6, 166) − binomcdf(250, .6, 133)

$=\boxed{.967}$

Jan 22-6:15 PM

I flipped a fair coin 400 times.
Success is to land tails.

1) $n = 400$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np = 200$ 5) $\sigma^2 = npq = 100$ 6) $\sigma = \sqrt{\sigma^2} = \sqrt{100} = 10$

7) Usual Range "95% Range" $\mu \pm 2\sigma$

$= 200 \pm 2(10)$

$\Rightarrow \boxed{180 \text{ to } 220}$

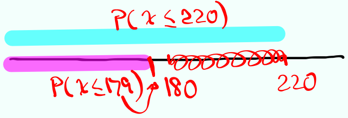
8) P(# of tails is between 180 to 220, inclusive)

Reduce by 1

$P(180 \leq x \leq 220) =$

binomcdf(400, .5, 220) − binomcdf(400, .5, 179)

$= \boxed{.960}$



Jan 22-6:24 PM

At a certain college, $\frac{1}{3}$ of students are international students.
I randomly selected 60 students.

1) $n = 60$ 2) $p = \frac{1}{3}$ 3) $q = \frac{2}{3}$

4) $\mu = np = 60(\frac{1}{3}) = \boxed{20}$ 5) $\sigma^2 = npq = 60(\frac{1}{3})(\frac{2}{3}) = \boxed{\frac{40}{3}}$

SG 16, Last Page, use fraction for P & q.

6) $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{40}{3}} \approx 3.651 \approx \boxed{4}$

7) 68% Range $\mu \pm \sigma = 20 \pm 4 \rightarrow \boxed{16 \text{ to } 24}$

8) P(# of international students are between 16 and 24, inclusive).

$P(16 \leq x \leq 24)$

$= \text{binomcdf}(60, \frac{1}{3}, 24) - \text{binomcdf}(60, \frac{1}{3}, 15)$

$\Rightarrow = \boxed{.783}$

SG 16 ✓

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SG 17-20

Continuous Random variable
with Prob. dist.

1) Uniform Prob. dist.

2) Standard Normal Prob. dist.

3) Normal Prob. dist.

4) Central limit theorem

5) Applications.

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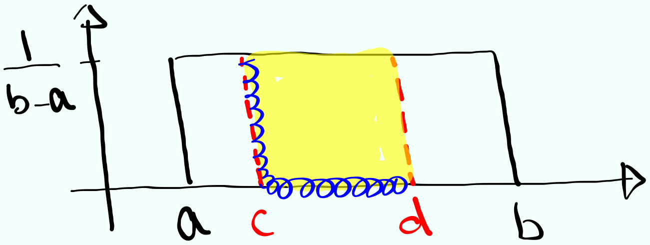
Uniform Prob. dist.

Let x be a Continuous random variable for all values from a to b with uniform Prob. dist.



$P(x=c)=0$

$P(c<x<d)=(d-c)\cdot\frac{1}{b-a}$

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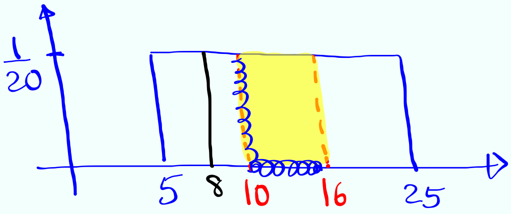
Consider a Uniform Prob. dist. for all values from 5 to 25.



$P(x=8)=0$

↑ Line

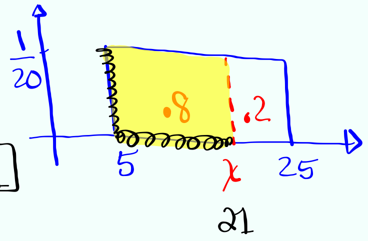
$P(10<x<16)$

$=(16-10)\cdot\frac{1}{20}=\frac{6}{20}=\boxed{.3}$

Find $x=P_{80}$

$(x-5)\cdot\frac{1}{20}=.8$

$x-5=20(.8)$ $\boxed{x=21}$



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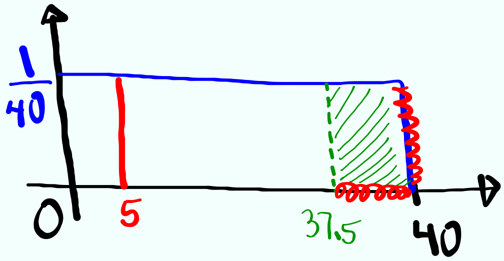
Consider a uniform Prob. dist. for all values from 0 to 40.



$P(x=5)=0$

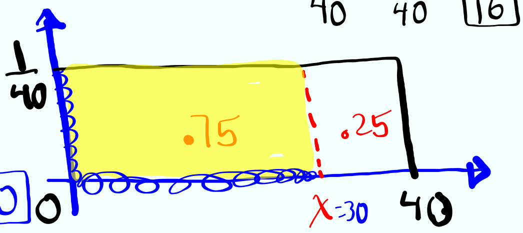
$P(x>37.5)=(40-37.5)\cdot\frac{1}{40}$

$=2.5\cdot\frac{1}{40}=\frac{2.5}{40}=\boxed{\frac{1}{16}}$

Find $x=Q_3$

$(x-0)\cdot\frac{1}{40}=.75$

$x=40(.75)=\boxed{30}$



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